Neki primeri izracunavanja granicne vrednosti:

1.

$$\lim_{x\to 1}\frac{x^2-\sqrt{x}}{\sqrt{x}-1}=\lim_{x\to 1}\frac{(\sqrt{x})^4-\sqrt{x}}{\sqrt{x}-1}=\lim_{x\to 1}\frac{\sqrt{x}((\sqrt{x})^3-1)}{\sqrt{x}-1}=$$

$$=\lim_{x\to 1}\frac{\sqrt{x}(\sqrt{x}\;\;-1)(1+\sqrt{x}+(\sqrt{x})^2)}{\sqrt{x}-1}=\lim_{x\to 1}\sqrt{x}(1+\sqrt{x}+x)=3$$

2.

$$\lim_{x\to 0}\frac{\sqrt{2+3x}-\sqrt{2-3x}}{x}=\lim_{x\to 0}\frac{\sqrt{2+3x}-\sqrt{2-3x}}{x}*\frac{\sqrt{2+3x}+\sqrt{2-3x}}{\sqrt{2+3x}+\sqrt{2-3x}}=$$

$$\lim_{x\to 0}\frac{2+3x-(2-3x)}{x(\sqrt{2+3x}-\sqrt{2-3x})}=\lim_{x\to 0}\frac{6x}{x(\sqrt{2+3x}+\sqrt{2-3x})}=$$

$$\lim_{x\to 0}\frac{6}{(\sqrt{2+3x}+\sqrt{2-3x})}=\frac{6}{\sqrt{2}+\sqrt{2}}=\frac{6}{2\sqrt{2}}=\frac{3\sqrt{2}}{2}$$

3.

$$\lim_{x\to 3}\frac{\sqrt{x+5}-\sqrt{2x+2}}{x^3-27}=\lim_{x\to 3}\frac{\sqrt{x+5}-\sqrt{2x+2}}{x^3-27}*\frac{\sqrt{x+5}+\sqrt{2x+2}}{\sqrt{x+5}+\sqrt{2x+2}}=$$

$$\lim_{x\to 3}\frac{x+5-(2x+2)}{(x^3-27)(\sqrt{x+5}+\sqrt{\cancel{2x}+2})}=\lim_{x\to 3}\frac{-(x-3)}{(x^3-27)(\sqrt{x+5}+\sqrt{2x+2})}=$$

$$=\lim_{x\to 3}\frac{-(x-3)}{(\cancel{x-3})(x^2+3x+9)(\sqrt{x+5}+\sqrt{2x+2})}=$$

$$\lim_{x\to 3}\frac{-1}{(x^2+3x+9)(\sqrt{x+5}+\sqrt{2x+2})}=\frac{-1}{27(2\sqrt{8})}=-\frac{1}{108\sqrt{2}}=-\frac{\sqrt{2}}{216}$$

4.

$$\lim_{x\to 4}\frac{2x-8\sqrt{x}+8}{\sqrt{x}-2}=\lim_{x\to 4}\frac{2(x-4\sqrt{x}+4)}{\sqrt{x}-2}=\lim_{x\to 4}\frac{2(\sqrt{x}-2)^2}{\sqrt{x}-2}=$$

$$\lim_{x\to 4}\frac{2(\cancel{\sqrt{x}-2})(\sqrt{x}-2)}{\cancel{\sqrt{x}-2}}=\lim_{x\to 4}\frac{2(\sqrt{x}-2)}{1}=0$$

5.

$$\lim_{x\to 8}\frac{\sqrt{x+8}-4}{2-\sqrt[3]{x}}=\lim_{x\to 8}\frac{\sqrt{x+8}-4}{2-\sqrt[3]{x}}*\frac{\sqrt{x+8}+4}{\sqrt{x+8}+4}=$$

$$\lim_{x\to 8}\frac{x+8-16}{(2-\sqrt[3]{x})(\sqrt{x+8}+4)}=\lim_{x\to 8}\frac{x-8}{(2-\sqrt[3]{x})(\sqrt{x+8}+4)}=$$

$$\lim_{x\to 8}\frac{-(8-x)}{(2-\sqrt[3]{x})(\sqrt{x+8}+4)}=\lim_{x\to 8}\frac{-(2^3-(\sqrt[3]{x})^3)}{(2-\sqrt[3]{x})(\sqrt{x+8}+4)}=$$

$$=\lim_{x\to 8}\frac{-(2-\sqrt[3]{x})(2^2+2\sqrt[3]{x}+(\sqrt[3]{x})^2))}{(2-\sqrt[3]{x})(\sqrt{x+8}+4)}=$$

$$= \lim_{x\to 8} \frac{-(\cancel{2-\sqrt[3]{x}})(2^2+2\sqrt[3]{x}+(\sqrt[3]{x})^2))}{(\cancel{2-\sqrt[3]{x}})(\sqrt{x+8}+4)} = \lim_{x\to 8} \frac{-(4+2\sqrt[3]{8}+(\sqrt[3]{8})^2))}{(\sqrt{8+8}+4)} =$$

$$= \frac{-(4+4+4)}{4+4} = -\frac{3}{2}$$

6.

$$\lim_{x\to 8} \frac{\sqrt[3]{8x}-\sqrt{x+8}}{x-8} = \lim_{x\to 8} \frac{2\sqrt[3]{x}-4+4-\sqrt{x+8}}{x-8} =$$

$$= \lim_{x\to 8}\left(\frac{2(\sqrt[3]{x}-2)}{x-8} - \frac{\sqrt{x+8}-4}{x-8} \right) =$$

$$= \lim_{x\to 8}\left(\frac{2(\sqrt[3]{x}-2)}{x-8} * \frac{(\sqrt[3]{x})^2+2\sqrt[3]{x}+4)}{(\sqrt[3]{x})^2+2\sqrt[3]{x}+4)} - \frac{\sqrt{x+8}-4}{x-8} * \frac{\sqrt{x+8}+4}{\sqrt{x+8}+4} \right) =$$

$$= \lim_{x\to 8}\left(\frac{2(x-8)}{(x-8)(\sqrt[3]{x})^2+2\sqrt[3]{x}+4)} - \frac{x+8-16}{(x-8)(\sqrt{x+8}+4)} \right) =$$

$$= \lim_{x\to 8}\left(\frac{2(x-8)}{(x-8)(\sqrt[3]{x})^2+2\sqrt[3]{x}+4)} - \frac{x+8-16}{(x-8)(\sqrt{x+8}+4)} \right) =$$

$$= \lim_{x\to 8}\left(\frac{2}{(\sqrt[3]{x})^2+2\sqrt[3]{x}+4)} - \frac{1}{(\sqrt{x+8}+4)} \right) = \frac{2}{4+4+4} - \frac{1}{4+4} = \frac{1}{24}$$

7.

$$\lim_{x\to -7}\frac{\sqrt[3]{x-1}+2}{x+7}=\lim_{x\to -7}\frac{\sqrt[3]{x-1}+2}{x+7}*\frac{(\sqrt[3]{x-1})^2-2\sqrt[3]{x-1}+4}{(\sqrt[3]{x-1})^2-2\sqrt[3]{x-1}+4}=$$

$$=\lim_{x\to -7}\frac{x-1+8}{(x+7)((\sqrt[3]{x-1})^2-2\sqrt[3]{x-1}+4)}=\frac{1}{(\sqrt[3]{-7-1})^2-2\sqrt[3]{-7-1}+4}=\frac{1}{12}$$

8.

$$\lim_{x\to 1}\frac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}=\lim_{x\to 1}\frac{\sqrt[3]{7+x^3}-2+2-\sqrt{3+x^2}}{x-1}=$$

$$=\lim_{x\to 1}\left(\frac{\sqrt[3]{7+x^3}-2}{x-1}-\frac{\sqrt{3+x^2}-2}{x-1}\right)=$$

$$=\lim_{x\to 1}\left(\frac{\sqrt[3]{7+x^3}-2}{x-1}*\frac{(\sqrt[3]{7+x^3})^2+2\sqrt[3]{7+x^3}+4}{(\sqrt[3]{7+x^3})^2+2\sqrt[3]{7+x^3}+4}-\frac{\sqrt{3+x^2}-2}{x-1}*\frac{\sqrt{3+x^2}+2}{\sqrt{3+x^2}+2}\right)=$$

$$=\lim_{x\to 1}\left(\frac{7+x^3-8}{(x-1)((\sqrt[3]{7+x^3})^2+2\sqrt[3]{7+x^3}+4)}-\frac{3+x^2-4}{(x-1)(\sqrt{3+x^2}+2)}\right)=$$

$$=\lim_{x\to 1}\left(\frac{(x-1)(x^2+x+1)}{(x-1)((\sqrt[3]{7+x^3})^2+2\sqrt[3]{7+x^3}+4)}-\frac{(x-1)(x+1)}{(x-1)(\sqrt{3+x^2}+2)}\right)=$$

$$=\lim_{x\to 1}\left(\frac{(x^2+x+1)}{((\sqrt[3]{7+x^3})^2+2\sqrt[3]{7+x^3}+4)}-\frac{(x+1)}{(\sqrt{3+x^2}+2)}\right)=\frac{3}{12}-\frac{2}{4}=-\frac{1}{4}$$